

FILL IN THE BLANKS using "a scalar", "a vector", or "undefined".

SCORE: ____ / 3 PTS

NOTE: You must get at least 2 out of 3 answers correct to earn any points.

If \vec{u} , \vec{v} and \vec{w} are vectors in three dimensions, then $(\vec{u} - \vec{v}) \times (\vec{u} \cdot \vec{w})$

is UNDEFINED,

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$(\vec{u} \cdot \vec{v})(\vec{u} \cdot \vec{w})$

is A SCALAR and

$(\vec{u} \times \vec{v})(\vec{u} \times \vec{w})$

is UNDEFINED.

In the diagram on the right, E is the midpoint of AC , and D is the midpoint of BE .

SCORE: ____ / 5 PTS

[a] Find an expression for vector \vec{AC} in terms of \vec{u} and \vec{v} .

$$\vec{AB} + \vec{BC} = \vec{u} + \vec{v} \quad \textcircled{\frac{1}{2}}$$

[b] Find an expression for vector \vec{BE} in terms of \vec{u} and \vec{v} .

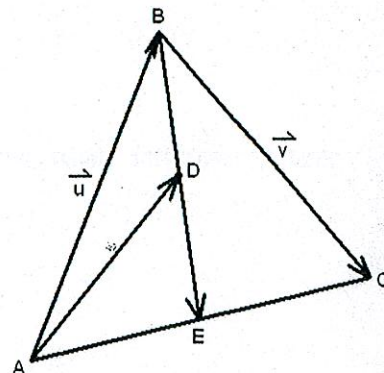
$$\vec{BA} + \vec{AE} = -\vec{AB} + \frac{1}{2}\vec{AC}$$

$$\textcircled{1} \quad -\vec{u} + \frac{1}{2}(\vec{u} + \vec{v}) = \frac{1}{2}\vec{v} - \frac{1}{2}\vec{u} \quad \textcircled{1}$$

[c] Find an expression for vector \vec{AD} in terms of \vec{u} and \vec{v} .

$$\vec{AB} + \vec{BD} = \vec{AB} + \frac{1}{2}\vec{BE}$$

$$\textcircled{\frac{1}{2}} \quad \vec{u} + \frac{1}{2}\left(\frac{1}{2}\vec{v} - \frac{1}{2}\vec{u}\right) = \frac{3}{4}\vec{u} + \frac{1}{4}\vec{v} \quad \textcircled{1}$$



Let P be the point $(-3, 2, -1)$, and Q be the point $(1, -2, -7)$.

SCORE: ____ / 5 PTS

[a] Find the symmetric equation of the line through P and Q .

DIRECTION VECTOR $\langle 4, -4, -6 \rangle$ or $\langle 2, -2, -3 \rangle$

$$\textcircled{1} \quad \frac{x+3}{2} = \frac{y-2}{-2} = \frac{z+1}{-3}$$

(OTHER VARIATIONS POSSIBLE)

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[b] Find the parametric equation of the line through Q and parallel to the line $\frac{6-z}{2} = 4+x = \frac{y-3}{5}$.

$$x = 1+t \quad \textcircled{1}$$

$$y = -2+5t \quad \textcircled{1} \quad \text{(OTHER VARIATIONS POSSIBLE)}$$

$$z = -7-2t \quad \textcircled{1}$$

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$$\frac{x-(-4)}{1} = \frac{y-3}{5} = \frac{z-6}{-2}$$

DIRECTION VECTOR $\langle 1, 5, -2 \rangle$

Use the properties of the cross product to simplify the expression $(\vec{b} + \vec{n}) \times (\vec{b} - 2\vec{n})$.

SCORE: ____ / 4 PTS

Write your final answer without negatives.

$$\begin{aligned} & \vec{b} \times \vec{b} + \vec{n} \times \vec{b} - \vec{b} \times 2\vec{n} - \vec{n} \times 2\vec{n} \\ &= \vec{0} + \vec{n} \times \vec{b} - 2(\vec{b} \times \vec{n}) - 2(\vec{n} \times \vec{n}) \\ &= \vec{n} \times \vec{b} + 2(\vec{n} \times \vec{b}) - 2(\vec{0}) = 3(\vec{n} \times \vec{b}) \end{aligned}$$

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Let $\vec{u} = \langle 2, -1, -3 \rangle$ and $\vec{q} = \langle -1, 4, 5 \rangle$.

SCORE: ____ / 13 PTS

[a] Find the (vector) projection of \vec{q} onto \vec{u} .

$$\begin{aligned} \frac{\vec{q} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} &= \frac{-2-4-15}{4+1+9} \langle 2, -1, -3 \rangle \\ &= \frac{-21}{14} \langle 2, -1, -3 \rangle \\ &= -\frac{3}{2} \langle 2, -1, -3 \rangle = \langle -3, \frac{3}{2}, \frac{9}{2} \rangle \end{aligned}$$

[b] Find the area of the triangle with vectors \vec{u} and \vec{q} as 2 of its sides.

$$\begin{aligned} \frac{1}{2} \|\vec{u} \times \vec{q}\| &= \frac{1}{2} \|\langle -5+2, -(10-3), 8-1 \rangle\| \\ &= \frac{1}{2} \|\langle -3, -7, 7 \rangle\| \\ &= \frac{1}{2} \sqrt{9+49+49} \\ &= \frac{1}{2} \sqrt{107} \end{aligned}$$

[c] Find the angle between the vectors \vec{u} and \vec{q} .

$$\begin{aligned} \cos^{-1} \frac{\vec{u} \cdot \vec{q}}{\|\vec{u}\| \|\vec{q}\|} &= \cos^{-1} \frac{-21}{\sqrt{14} \sqrt{42}} \\ &= \cos^{-1} \frac{-7 \cdot 3}{\sqrt{2} \sqrt{7} \sqrt{2} \sqrt{3} \sqrt{7}} \\ &= \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) = \frac{5\pi}{6} \text{ or } 150^\circ \end{aligned}$$

[d] Find a unit vector perpendicular to both \vec{u} and \vec{q} .

$$\frac{\vec{u} \times \vec{q}}{\|\vec{u} \times \vec{q}\|} = \frac{\langle -3, -7, 7 \rangle}{\sqrt{107}} = \left\langle -\frac{3}{\sqrt{107}}, -\frac{7}{\sqrt{107}}, \frac{7}{\sqrt{107}} \right\rangle$$